Natural spectral line width :->

It arises from the uncertainty in energy of the states involved in the transitions

If ΔE is the energy width from which the electron jump and spectrum line is emitted.
let τ_i is mean life time of this state, then

$$\boxed{\Delta E \, \Delta\tau_i \simeq \hbar}$$

uncertainty principle.

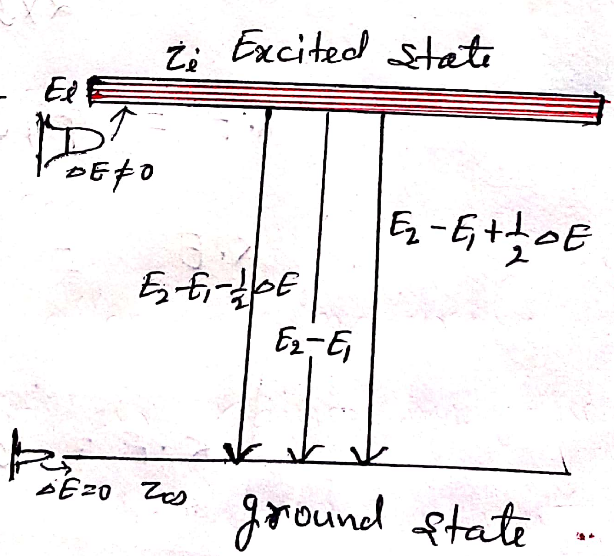
$$\Delta E = h \, \Delta\nu$$

$$h \, \Delta\nu \, \Delta\tau_i \simeq \frac{h}{2\pi}$$

$$\Delta\nu_N \simeq \frac{1}{2\pi \, \Delta\tau_i}$$

$$2\pi \, \Delta\nu_N \simeq \frac{1}{\Delta\tau_i}$$

$$\boxed{\Delta\omega_N \simeq \frac{1}{\Delta\tau_i}}$$



Full width half maxima (FWHM) / Half width / Natural width we obtained the natural line width ($\Delta\nu_N$) of a transition from level $|i\rangle$ to the ground state $|0\rangle$.

$$\Delta\omega_N = A_i = \frac{1}{\tau_i}$$

where A_i = Einstein Coefficient for Spontaneous Emission.

Life time of an excited state is about

$$\tau = 10^{-8} \text{ sec.}$$

for $\tau = 10^{-8}$ sec.

$$\Delta\nu_N = \frac{1}{2\pi \times 10^{-8}}$$

$$\Delta\nu_N = 10^{7} \text{ sec}^{-1}.$$

This half width is called the natural line width because it is caused by the Spontaneous emission of the atom without any external influence.

For a transition between two excited states E_K and E_i with life time τ_K and τ_i then

$$\Delta E = \Delta E_i + \Delta E_K.$$

$$\boxed{\Delta\nu_N = \frac{1}{2\pi}\left(\frac{1}{\tau_i} + \frac{1}{\tau_K}\right)}$$